MATH1010I Quiz 2 Date: 6-11-2014 Time Limit: 45 minutes Total: 20 marks

Name (in print): _____ (Student ID: _____)

- Answer ALL questions.
- Provide steps in your calculations and/or give appropriate justification for your answers.

1. (6 marks)
Consider the curve $\exp(2x)y - 2x\ln(y) - 1 = 0$. Note that $(0,1)$ is a point on this curve.

Consider the function y of x implicitly defined by this curve at and near the point $(0,1)$.

(a) Show that $\left(\exp(Ax) + \dfrac{Bx^C}{y^D} + E\right)\dfrac{dy}{dx} + (F\exp(Gx)y^H + K\ln(y) + L) = 0$.

Here $A, B, C, D, E, F, G, H, K, L$ are rational numbers, whose respective values you have to determine.

(b) Compute $\left.\dfrac{dy}{dx}\right|_{x=0,y=1}$ and $\left.\dfrac{dx}{dy}\right|_{x=0,y=1}$.

(c) Compute $\left.\dfrac{d^2y}{dx^2}\right|_{x=\frac{\pi}{2},y=\pi}$.

2. (6 marks)
Define the function $f : [0,+\infty) \longrightarrow \mathbb{R}$ by $f(x) = (x^2 - 5x + 5)\exp(-x)$ for any $x \in [0,+\infty)$. Note that f is continuous on $[0,+\infty)$ and is differentiable on $(0,+\infty)$.

(a) Determine the value of $\lim\limits_{x\to+\infty} f(x)$ if the limit exists.

(b) Compute $f'(x)$ on $(0,+\infty)$.

(c) Determine all local extrema and all absolute extrema, if any, of f on $[0,+\infty)$.

3. (8 marks)
Evaluate each of the limits below. Where appropriate and necessary, you may apply L'Hôpital's Rule. Mark (H) under the equality signs where you apply L'Hôpital's Rule.

(a) $\lim\limits_{x\to0^+} \dfrac{\ln(\sin(3x))}{\ln(\sin(2x))}$

(b) $\lim\limits_{x\to0^+} \left(\dfrac{1}{\sin(x)} - \dfrac{1}{x - x^2}\right)$

(c) $\lim\limits_{x\to0^+} (x + \sin(x))^x$

(d) $\lim\limits_{x\to+\infty} \dfrac{x^2 + \cos(e^x)}{x^2 + \cos(e^{2x})}$

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